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PORTFOLIO DECISION MODEL BASED ON FUZZY EXCESS RETURN

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ABSTRACT

Due to the existence of the uncertainty of risks and returns in China's financial securities market, we construct a portfolio model by applying different measuring method and considering restriction of various factors on investors in securities market. Based on the framework of traditional rational investors' portfolio theory, triangular fuzzy number is employed to characterize the excess return on securities. In our model, downside risk measured by measuring lower semi-deviation is used as the risk index to measure excess return，and coefficient of variation(CV) works as the standard to measure the quality of each stock in order to gain the optimal portfolio. At last, an empirical analysis of SSE 50 Index implies that our portfolio decision model based on fuzzy excess return has higher excess return compared with other traditional portfolio decision theory.

Keywords: *excess return; triangular fuzzy number; portfolio decision.*

I. INTRODUCTION

Nowadays, uncertainty exists in Chinese security market as it exist in others market. So far, there are few researches on fuzzy uncertainty of return on investment and risk measure, while most researchers conduct studies simply from the perspective of randomness. For example, Markowitz[1] established security investment theory by using quantitative method. He applied mean-variance method to the calculation of expected return rate of investors and the possible risk, and obtained the efficient frontier. SMO algorithm, introduced by Zhang[2], can be used to maximum the efficiency of portfolio under certain mean-variance and select the optimal portfolio and it can be widely applied to portfolio in different sizes. Obviously, these traditional models have limitation. Firstly, uncertainty of current security market contains two parts, randomness existing in most markets and fuzziness resulted from society, culture, economy and people’s mind. According to information sciences, randomness often involves only the quantification of certain information, while fuzziness concerns the meaning of the whole information. Secondly, people always regard the expected return and risk as random variables, which can be gained by calculating the expectation and variance based on the historical statistic. In fact, investors pay more attention to the assets' future earnings potential rather than its past performance. Thirdly, there are a number of problems leading to the uneven distribution of information at the initial stage of development of Chinese security market, including the weakness of quality of information publication and insufficiency of information. According to this imperfect information, deficiencies in decisions made by investors based on the traditional investment decision-making method are obvious and make the traditional method become unrealistic. Later, Li and Xu[3] introduced the establishment of portfolio, combining the fuzzy set theory, mentioned in Zadeh’s[4] paper, with the variables introduced by Kwakernaak[5]. This investment model is much more realistic and it is proved that it can get a better modern portfolio. In order to overcome the deficiency of using variance to measure risk, in 1970, Mao[6] and Swalm[7] used semi-variance to measure risk and established the mean-semi-variance portfolio model. The method of measuring risk with lower semi-deviation is much more realistic, as the downside risk is a more accurate measure of the risk in the stock market, which is more in line with people's psychology. In terms of return, alpha strategy, developed from Jason index, also called alpha index, is in line with our country, an emerging market. Compared with traditional arbitrage methods, the return from the alpha strategy depends on the investors’ ability of finding excess returns which means it cannot be easily replicated. Therefore, this paper aims to optimize the excess return rate with fuzzy theory, which does great help to improve the traditional investment screening process and establish a more perfect investment model. Meanwhile, we employed the cardinality constraints of portfolio, the boundary constraints of investment ratio and the degree of diversification, proposed by Liu[8], to our model. Besides, transaction costs and loan constraints are added in order to establish a multi-criteria portfolio optimization model. This model not only has great academic value, but also has strong practical significance, because it considers the main practical problems faced in the current Chinese security market.

Overall, this paper employed the fuzzy theory to establish a multi-criteria portfolio optimization model, considering Chinese security market's practical constraints, in order to simulate the transaction activity in the real market. Then the weighted fuzzy objective programming method will be applied to transform the problem into a single objective programming problem, aiming to design an algorithm to solve the model. This algorithm greatly improves the efficiency of the portfolio decision-making, and we can gain a reasonable proportion of the allocation of securities investment and a certain excess return rate.

II. FUZZY EXCESS RETURN MODEL

2.1 Excess Return α

Excess return, known as "alpha", is a portfolio return minus the market return. In the classical Capital Asset Pricing Model, supposing $\beta = \sigma_{im}/{\sigma_m}^2$, R_i, where is the Security Expected Rate of Return, equal to $R_f + \beta(R_m - R_f)$, is derived only based on the market factors. However, we find that there are always intercept terms in the formula while conducting the actual back-testing, which are resulted from other factors besides market factors. And these intercept terms are regarded as excess return. Therefore, based on the classical Capital Asset Pricing Model, in order to get excess return α, this dissertation will carry out unified regression of week market return using the week market return of the last year. So αcan be regarded as the indicator value of securities future value.

2.2 The Application ofTriangular Fuzzy Number

The above derivation of excess return α is calculated according to history data. However, in the actual stock market, there is always difference between history data and future data. Furthermore, given investors' relatively subjective selection of the stock, the excess return derived according to the history data can only be regarded as a reference of future excess return.

In order to improve the precision of the measurement of securities' future excess return, this dissertation introduces Zadeh's fuzzy set theory to construct triangular fuzzy number with proper extend, so that the securities' future excess return can be represented.

According to the related principles of triangular fuzzy number, the construction of triangular fuzzy number of each stock must meet following conditions:

(1) For $\alpha \in (0,1]$, $\alpha(A)$ must be a closed interval (also said a convex set) ;

(2) The underlying setof A must be bounded;

(3) The membership function must meet following formulas:

$$\mu(x) = \begin{cases} \frac{x}{\alpha-l} - \frac{l}{\alpha-l} x \in [l,\alpha] \\ \frac{x}{\alpha-u} - \frac{u}{\alpha-u} x \in [\alpha,u] \\ 0 \qquad otherwise \end{cases} \quad (1)$$

where α is the excess return derived according to the history data, l is the lower limit of fuzzy number, u is the upper limit of fuzzy number.

2.3 The selections of the upper and lower limits of fuzzy number

In consequence of the fact that the upper and lower limits of future excess return fuzzy number is uncertain, this dissertation selects 85% quantile of the history excess return, as the upper bound as well as 15% quantile as the lower bound.

$$\begin{cases} F(u) = 0.85; \quad u \in (C_1, C_2, C_3, \dots \dots, C_i) \\ F(l) = 0.15; \quad l \in (C_1, C_2, C_3, \dots \dots, C_i) \end{cases} \quad (2)$$

where $C_i = r_i - r_{mi}$, $i = 1, 2, 3, \dots \dots$, which is the difference between history stock returns and market return.

2.4 The measure of the risk of stocks excess return

As it is impossible to compare the stock returns and risks among each stock by applying the triangular fuzzy numbers, this dissertation will introduce modified coefficient of variation as the standard of stock measurement. Besides, since investors prefer higher-than-average returns, the semi-variance is employed for a measure of the risk of excess return.

Construct the following computing formulas:

(1) Define F is the set of all fuzzy number in the context of real number;

(2) Assume A ∈ F, $[A]\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)]$, where ($\gamma \in [0,1]$), $[A]\gamma$ is the level set under γ degree of membership, and $\underline{a}(\gamma) = l + \gamma(F_m(x) - l)$, $\overline{a}(\gamma) = u - \gamma(u - F_m(x))$, therefore, the mean value of fuzzy numbers can be expressed as:

$$E(A) = \int_0^1 \gamma(\underline{a}(\gamma) + \overline{a}(\gamma))d\gamma \; ; \qquad (3)$$

(3) Lower partial semi-variance is expressed as:

$$Var^-(A) = 2\int_0^1 \gamma\,(E(A) - \underline{a}(\gamma))^2 d\gamma \; ; \qquad (4)$$

(4) Modified coefficient of variation is expressed as:

$$CV = \frac{Var^-(A)}{E(A)} \; ; \qquad (5)$$

Modified coefficient of variation measures the risk of stocks excess return by measuring the risk of returns lower than average. The smaller the modified coefficient of variation value is, the wider range of possibilities of gaining continuous excess return is.

2.5PortfolioModel

Utilizing means and variance, Markowitz(1952) built portfolio model for rational investors under uncertain environment. However, there are some differences when fuzzy theory is introduced to the model, so basing on the means-variance portfolio model and reality constraints, the modified portfolio model is showed below. To make it easier to understand the following model, all the notations which will be used in the model are listed as follows:

x_i the investment proportion of stock i, where $x_i \geq 0$

x the investment matrix, where x = $(x_1, x_2, x_3, \ldots)'$

x_f the investment proportion of risk-free asset,where $x_f = 1 - \sum_{i=1}^{n} x_i$

$A_i$the fuzzy excess return of stock i, where $A_i = (\alpha_i, l_i, r_i)$

W_0 initial capital

r_e the weekly excess return for the forecasting week

l_{i0} the minimum investment proportion of stock i

$u_{i0}$the maximum investment proportion of stock i

K the maximum number of assets in the portfolio, whererank xi = K, that means the top K number of stocks which have low CV value will be chosen.

C_t transaction fees,which is $C_t = W_0 \cdot 0.003 + W_t \cdot 0.004$ (Note: the stamp tax is 1‰,the commission fee is 3‰ and the transfer fee is ignored)

E(A) the matrix of $E(A_i)$, where $E(A)=(E(A_1), E(A_2), E(A_3), \ldots)'$

So for any two stocks, the equation of the lower partialsemi-covariance will be

$$Cov^-(A_i, A_j) = 2\int_0^1 \gamma[E(A_i) - a_i(\gamma)][E(A_j) - a_j(\gamma)]\, d\gamma$$

And for all stocks, the matrix of the lower partial semi-covariance is

$$Cov^- = \begin{pmatrix} Cov_{11}^- & Cov_{12}^- & Cov_{13}^- & \ldots \\ Cov_{21}^- & Cov_{22}^- & Cov_{23}^- & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

At the end, the portfolio decision model showed below.

$$\text{Max } [E(A)'x - x'Cov^- x] - r_f \cdot x_f \qquad (6)$$

s.t.

$$\begin{cases} x_f + \sum_{i=1}^{n} x_i = 1 \\ W[E(A)' \cdot x + r_f \cdot x_f] \geq C_t \\ l_{i0} \leq x_i \leq u_{i0} \\ K = 5 \end{cases}$$

For each stock in this paper, the maximum investment proportion is 100% while the minimum is 0%, so l_i0 = 0,u_{i0} = 1.Meanwhile, the weekly excess return for the forecasting week is $re = r * x + r_f * x_f - r_m$, where $r = (r_1, r_2, \ldots, r_i)$ which is the weekly return of stock, and r_m is return of the market.

III. EMPIRICAL ANALYSIS

Basingtheweeklydata, the portfoliodecisionmodel that wasbuiltabovetocalculatethe ratios of the optimalportfolio is used,whichdecidesthe investment ratios for different stock in that forecasting week. The data which includes SSE 50 Index (000016.SH) and its constituent stockfrom Wind platform from April 28, 2015 to July 29, 2016 will be

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collected and used below. Meanwhile, five-year Treasury yields, which was 4.17%, will be chosen as risk-free interest rate, so the weekly risk-free interest rate will be 0.008%.

Taking the week from April 18, 2016 as an example. Basing method in chapter 2.1, the intercept terms of 50 SSE 50 Index constituent stocks was obtained by using traditional CAPM model. The outcomes are showed in the following Table 1.

Table 1

Stock Code	Intercept Term	Stock Code	Intercept Term
600000.SH	0.0058	601766.SH	-0.0126
600104.SH	0.0058	600887.SH	0.0037
600050.SH	-0.0042	601688.SH	0.0027
600036.SH	0.0055	600518.SH	0.0058
600030.SH	-0.0014	600999.SH	-0.0025
600028.SH	-0.0023	600637.SH	-0.0044
600016.SH	0.0043	601998.SH	0.0053
600519.SH	0.0067	600109.SH	-0.0002
601006.SH	-0.0052	600893.SH	0.0051
601398.SH	0.0015	600958.SH	0.0044
600048.SH	0.0036	601800.SH	0.0083
601628.SH	-0.0002	601988.SH	0.0023
601166.SH	0.0046	601186.SH	0.0021
601318.SH	0.0009	601390.SH	-0.0016
601328.SH	0.0046	601336.SH	0.0029
601088.SH	0.0049	601211.SH	-0.0022
601857.SH	-0.0033	601985.SH	0.0124
601601.SH	0.0030	601377.SH	-0.0015
600837.SH	-0.0016	601788.SH	0.0008
601169.SH	0.0050	600029.SH	0.0036
601668.SH	0.0014	600100.SH	0.0028
601288.SH	0.0028	600485.SH	-0.0067
600111.SH	0.0027	600547.SH	0.0109
601818.SH	0.0034	601901.SH	-0.0005
601989.SH	-0.0060	601198.SH	0.0088

Then, the 15% quantile and 85% quantile of each stock's excess return in the last year are introduced as lower bound and upper bound of fuzzy excess rate. Both of quantiles are showed in the following Table 2.

Table 2

Stock Code	15% Quantile	85% Quantile	Stock Code	15% Quantile	85% Quantile
600000.SH	-0.0279	0.0495	601766.SH	-0.0364	0.0158
600104.SH	-0.0229	0.0264	600887.SH	-0.0161	0.0351
600050.SH	-0.0365	0.0296	601688.SH	-0.0499	0.0394
600036.SH	-0.0147	0.0389	600518.SH	-0.0397	0.0644
600030.SH	-0.0417	0.0312	600999.SH	-0.0420	0.0361
600028.SH	-0.0238	0.0206	600637.SH	-0.0632	0.0522
600016.SH	-0.0244	0.0401	601998.SH	-0.0263	0.0210
600519.SH	-0.0228	0.0419	600109.SH	-0.0600	0.0865
601006.SH	-0.0293	0.0147	600893.SH	-0.0625	0.0705
601398.SH	-0.0269	0.0357	600958.SH	-0.0533	0.0450
600048.SH	-0.0281	0.0434	601800.SH	-0.0558	0.0553
601628.SH	-0.0347	0.0277	601988.SH	-0.0251	0.0289
601166.SH	-0.0150	0.0287	601186.SH	-0.0550	0.0388

601318.SH	-0.0143	0.0244	601390.SH	-0.0607	0.0432
601328.SH	-0.0248	0.0241	601336.SH	-0.0330	0.0406
601088.SH	-0.0320	0.0237	601211.SH	-0.0378	0.0303
601857.SH	-0.0286	0.0192	601985.SH	-0.0442	0.0490
601601.SH	-0.0219	0.0344	601377.SH	-0.0554	0.0411
600837.SH	-0.0400	0.0320	601788.SH	-0.0552	0.0488
601169.SH	-0.0293	0.0402	600029.SH	-0.0649	0.0563
601668.SH	-0.0275	0.0203	600100.SH	-0.0755	0.0641
601288.SH	-0.0243	0.0373	600485.SH	-0.0842	0.0838
600111.SH	-0.0485	0.0419	600547.SH	-0.0538	0.0915
601818.SH	-0.0181	0.0170	601901.SH	-0.0459	0.0323
601989.SH	-0.0517	0.0579	601198.SH	-0.0857	0.1006

Utilizing the method in chapter 2.2 and chapter 2.3, the fuzzy excess return of each stock can be produced as the following Table 3.

Table 3

Stock Code	Fuzzy Excess Return	Stock Code	Fuzzy Excess Return
600000.SH	(0.0058,-0.0279,0.0495)	601766.SH	(-0.0126,-0.0364,0.0158)
600104.SH	(0.0058,-0.0229,0.0264)	600887.SH	(0.0037,-0.0161,0.0351)
600050.SH	(-0.0042,-0.0365,0.0296)	601688.SH	(0.0027,-0.0499,0.0394)
600036.SH	(0.0055,-0.0147,0.0389)	600518.SH	(0.0058,-0.0397,0.0644)
600030.SH	(-0.0014,-0.0417,0.0312)	600999.SH	(-0.0025,-0.0420,0.0361)
600028.SH	(-0.0023,-0.0238,0.0206)	600637.SH	(-0.0044,-0.0632,0.0522)
600016.SH	(0.0043,-0.0244,0.0401)	601998.SH	(0.0053,-0.0263,0.0210)
600519.SH	(0.0067,-0.0228,0.0419)	600109.SH	(-0.0002,-0.0600,0.0865)
601006.SH	(-0.0052,-0.0293,0.0147)	600893.SH	(0.0051,-0.0625,0.0705)
601398.SH	(0.0015,-0.0269,0.0357)	600958.SH	(0.0044,-0.0533,0.0450)
600048.SH	(0.0036,-0.0281,0.0434)	601800.SH	(0.0083,-0.0558,0.0553)
601628.SH	(-0.0002,-0.0347,0.0277)	601988.SH	(0.0023,-0.0251,0.0289)
601166.SH	(0.0046,-0.0150,0.0287)	601186.SH	(0.0021,-0.0550,0.0388)
601318.SH	(0.0009,-0.0143,0.0244)	601390.SH	(-0.0016,-0.0607,0.0432)
601328.SH	(0.0046,-0.0248,0.0241)	601336.SH	(0.0029,-0.0330,0.0406)
601088.SH	(0.0049,-0.0320,0.0237)	601211.SH	(-0.0022,-0.0378,0.0303)
601857.SH	(-0.0033,-0.0286,0.0192)	601985.SH	(0.0124,-0.0442,0.0490)
601601.SH	(0.0030,-0.0219,0.0344)	601377.SH	(-0.0015,-0.0554,0.0411)
600837.SH	(-0.0016,-0.0400,0.0320)	601788.SH	(0.0008,-0.0552,0.0488)
601169.SH	(0.0050,-0.0293,0.0402)	600029.SH	(0.0036,-0.0649,0.0563)
601668.SH	(0.0014,-0.0275,0.0203)	600100.SH	(0.0028,-0.0755,0.0641)
601288.SH	(0.0028,-0.0243,0.0373)	600485.SH	(-0.0067,-0.0842,0.0838)
600111.SH	(0.0027,-0.0485,0.0419)	600547.SH	(0.0109,-0.0538,0.0915)
601818.SH	(0.0034,-0.0181,0.0170)	601901.SH	(-0.0005,-0.0459,0.0323)
601989.SH	(-0.0060,-0.0517,0.0579)	601198.SH	(0.0088,-0.0857,0.1006)

After obtaining all the fuzzy excess return, the modified CV model mentioned in chapter 2.4 is applied to calculate CV value of each stock.

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Table 4

Stock Code	CV	Stock Code	CV
600000.SH	0.0307	601766.SH	-0.0090
600104.SH	0.0253	600887.SH	0.0168
600050.SH	-0.0456	601688.SH	5.0385
600036.SH	0.0133	600518.SH	0.0522
600030.SH	-0.0891	600999.SH	-0.0961
600028.SH	-0.0387	600637.SH	-0.1181
600016.SH	0.0294	601998.SH	0.0447
600519.SH	0.0215	600109.SH	0.1868
601006.SH	-0.0146	600893.SH	0.1580
601398.SH	0.0619	600958.SH	0.2914
600048.SH	0.0398	601800.SH	0.1048
601628.SH	-0.1361	601988.SH	0.0569
601166.SH	0.0138	601186.SH	-0.3281
601318.SH	0.0239	601390.SH	-0.1244
601328.SH	0.0388	601336.SH	0.0697
601088.SH	0.0854	601211.SH	-0.0739
601857.SH	-0.0264	601985.SH	0.0461
601601.SH	0.0299	601377.SH	-0.1250
600837.SH	-0.0933	601788.SH	-0.8415
601169.SH	0.0386	600029.SH	0.7070
601668.SH	-0.4201	600100.SH	-15.6120
601288.SH	0.0364	600485.SH	-0.2456
600111.SH	0.5091	600547.SH	0.0603
601818.SH	0.0284	601901.SH	-0.1103
601989.SH	-0.1530	601198.SH	0.1746

Basing on the Table 4, as for a stock, the lower CV value means the better performance, so all stocks are ranked from the lowest CV value to the highest CV value. And the top 5 stocks in the list are chosen and put into the investment pool. The detail of the stocks showed in the Table 5.

Table 5

Rank	Stock Code
NO.1	600036.SH
NO.2	601166.SH
NO.3	600887.SH
NO.4	600519.SH
NO.5	601318.SH

According to the equation $Cov^{-}(A_i, A_j)=2\int_0^1 \gamma[E(A_i) - a_i(\gamma)][E(A_j)-a_j(\gamma)]\,d\gamma$, the Cov- matrix of these five stocks can be got. The calculate result is showed in the following Table 6.

Cov^{-} matrix: $*10^{-4}$

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Table 6

	600036.SH	601166.SH	600887.SH	600519.SH	601318.SH
600036.SH	1.0225	0.8695	0.9849	1.2934	0.7454
601166.SH	0.8695	0.7436	0.8382	1.1067	0.6347
600887.SH	0.9849	0.8382	0.9487	1.2469	0.7181
600519.SH	1.2934	1.1067	1.2469	1.6473	0.9442
601318.SH	0.7454	0.6347	0.7181	0.9442	0.5436

In the end, utilizing portfolio selection model in chapter 2.5, the investment proportion of the portfolio was computed, which is showed in Table 7.

Table 7

Stock Code	Proportion
600036.SH	0.5284
601166.SH	0.0963
600887.SH	0.0968
600519.SH	0.1008
601318.SH	0.0902
Treasury Bonds	0.0874

According to the proportion in Table 7, 2.57% weekly excess return can be gained in that week after doing some investment. Then, basing on the same method, we tried to test this model in long-term and we continued to choose the SSE 50 Index constituent stocks as our selection stocks. The test period is from April 18, 2016 to July 29, 2016 and all the weekly excess return will be accumulate to plot an excess return curve. Finally, 12.23% excess return was gained at the end of the test. The curve of the back testing is showed in the Figure 1.

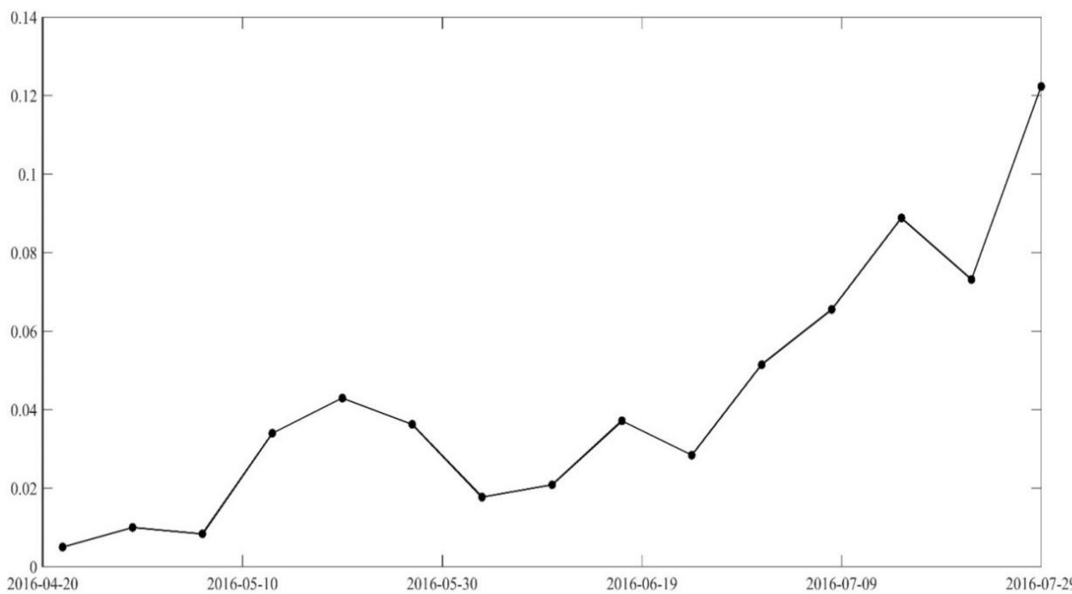

Figure 1

IV. CONCLUSION

Contraposed that the traditional portfolio decision ignores some factors like social, culture, economic and mentality of people in the financial market in China, this passage put forward to optimize the excess return base on the fuzzy theory, cite lower partial semi-variance as the risk indicator at the same time to optimize the screening process in the traditional portfolio, to build a better portfolio decision model. To build a multiple-criteria portfolio optimization model, we add constraint of cardinality and decentralization, boundary constraint of investment proportion, transaction expenses and loan constraint into consideration. As for the problem of the model solving, this passage transforms it into a single target programming problem by using weighted fuzzy target programming. Therefore, we design an algorithm to solve the model. In this way, we can not only solve the model problem with high efficiency, but also obtain a reasonable proportion of securities investment and excess return. We can draw a conclusion that, through the empirical analysis done on the Shanghai Stock Exchange 50, the portfolio decision model which based on fuzzy excess return can reach a higher excess return than traditional portfolio decision model.

But at the same time, there are still some deficiencies. Due to the uncertainty of the emotional fluctuation in the market, the using of 15% and 85% quintile as extended triangle fuzzy number in this passage cannotdescribe the uncertainty accurately so that in long-period practice, there are some excess returns become negative. How to measure a more accurate extend triangle fuzzy number will be the direction of subsequent research.

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