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## CHANNEL ESTIMATION OF MIMO-OFDM SYSTEM USING LS, MMSE, SBEM, LINEAR AND SPLINE

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### ABSTRACT

A major challenge to MIMO-OFDM systems is how to obtain the channel state information accurately and promptly for coherent detection of information symbols and channel synchronization. estimation problem for MIMO-OFDM systems and proposes a pilot-tone based estimation algorithm. A complex equivalent baseband MIMO-OFDM signal model is presented by matrix representation. By choosing  $L$  equally-spaced and equally-powered mpilot tones from  $N$  sub-carriers in one OFDM symbol, a down-sampled version of the original signal model is obtained. Furthermore, this signal model is transformed into a linear form solvable for the LS (least-square) estimation algorithm.  $y$ . In this paper an extensive review on different channel estimation methods used in MIMO-OFDM like pilot based, least square (LS) and minimum mean square error method (MMSE), SBEM, Linear and Spline methods and also other channel estimation methods used in MIMO-OFDM are discussed

**KEY— massive MIMO, OFDM, LS, MMSE, SBEM, inear and Spline.**

### INTRODUCTION

Wireless communications widely used for applications like a TV, Radio broadcasting and personal communication mobile, pager, cordless phones to cellular phones and satellite-based TV systems. Today wireless communication systems based mobile communication used broadband 3G and 4G high speed network to access internet services. In high speed network 4G networks best out of other 3G and broadband network, the 4G network based on OFDM system. The reason of choosing Orthogonal Frequency Division Multiplexing (OFDM) and Multiple Input Multiple Output (MIMO) both technologies essential parts of advanced wireless communications systems needs of high data rate, efficient spectrum utilization and reliable error-free communication. In M.I.M.O-O.F.D.M system channel estimation required using different levels of impact to synchronization and sub-carrier allocation used in channel estimation technique. A comparative analysis on popular pilot assisted estimation algorithms is provided, including LS, LMMSE pilot estimation algorithms. In thesis introduction discusses fundamentals of MIMO and OFDM systems Firstly, we will introduce the basic concepts of OFDM and MIMO systems as well Secondly, different channel estimation techniques are discussed. Finally, the data detection for MIMO systems is covered, which can be naturally extended to other systems such as OFDM systems over time-varying channel, cooperative MIMO systems

### SYSTEM MODEL AND PROBLEM FORMULATION

#### MIMO-OFDM System Model

A MIMO-OFDM system model is depicted in Figure 1.1. The system has  $N_T$  transmitting (TX) antennas,  $N_R$  receiving (RX) antennas and  $K$  sub-carriers in one OFDM block. It is assumed that time-variant wireless channel obey Rayleigh distribution and is quasi-static in consecutive  $P$  OFDM block duration. The maximum multipath delay length is  $L$ . The length of Cyclic Prefix (CP) is chosen to be longer than  $L$ . Channels between couples of TX-RX antennas are mutual uncorrelated. At a transmission time  $n$ , a stream of binary bits  $b$  is coded into  $NT$  symbol blocks. Then the signal on  $k$ th sub-carrier at  $i$ th TX antenna is denoted by  $X_i[n, k]$ , where  $i = 1, \dots, N_T$ ,  $k = 0, \dots, K - 1$ ,  $n = 0, \dots, P - 1$ . The received signal at RX antenna  $j$  is  $Y_j[n, k]$

$$Y_j[n, k] = \sum_{i=1}^{N_T} X_i[n, k]H_{ij}[n, k] + N_j[n, k] \quad 1.1$$

Where  $H_{ij}[n, k]$  is the frequency response between antennas  $i$  and  $j$ ,  $N_i[n, k]$  is the additive Gaussian noise with zero mean and variance  $\sigma_n^2$ . Since RX antennas are merely replicas to each other, the notation  $j$  for RX antenna will be omitted in the following part. Thus, for convenience of later mathematical manipulation,

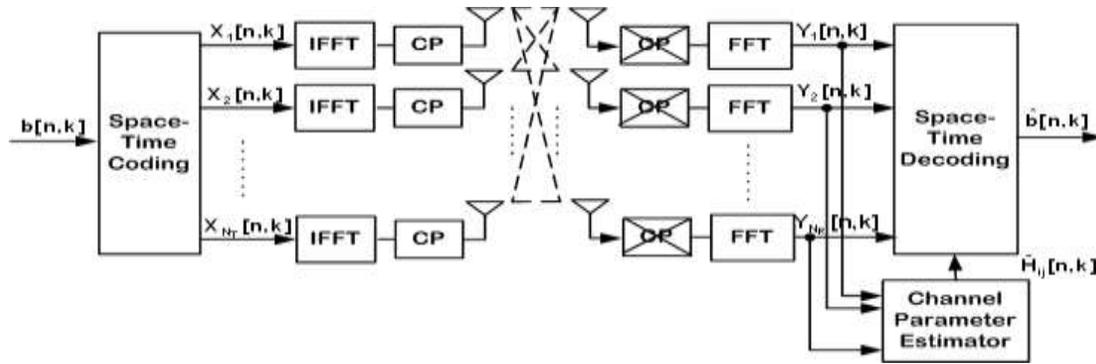


Figure 1. MIMO-OFDM System with channel estimator

Then Eq.(1.1) can be rewritten as

$$Y(n) = X(n)H + N(n) \quad 1.2$$

Considering the case of  $P$  blocks of pilot symbols, there are

$$\begin{aligned} Y &= [Y^T(0) Y^T(1) \dots Y^T(P-1)]^T \in \mathbb{C}^{KP \times 1} \\ X &= [X^T(0) X^T(1) \dots X^T(P-1)]^T \in \mathbb{C}^{KP \times KN} \\ N &= [N^T(0) N^T(1) \dots N^T(P-1)]^T \in \mathbb{C}^{KP \times 1} \end{aligned} \quad 3.4$$

Thus the system equation in frequency domain comes to be

$$Y = XH + N \quad 1.3$$

For OFDM systems with proper cyclic extension and sample timing, it has been shown in that, with tolerable leakage, the channel frequency response can be expressed as the Discrete Fourier Transform (DFT) of received time domain impulses like

$$H_i[n, k] = \sum_{l=0}^{L-1} h_t[n, l] W_k^{kl} \quad 1.5$$

where  $W_k = \exp(-j(2\pi/K))$ ,  $k$  is the number of sub-carriers (tones) of an OFDM block. The average power of  $h_i[n, l]$  and index  $L \ll K$  depend on the delay profiles and dispersion of the wireless channels. Specifically, time domain channel response from TX antenna  $i$  can be represented in vector format:

$$\begin{aligned} h_i &= [h_i(0) h_i(1) \dots h_i(L-1)]^T \\ h &= [H_1^T H_2^T \dots H_{N_T}^T]^T \in \mathbb{C}^{N_T \times L} \end{aligned} \quad 1.6$$

If we define the  $K \times K$  DFT transform matrix  $\mathbf{F}\mathbf{K}$  and  $K \times L$  mapping matrix  $\mathbf{M}$  for zero-padding as

$$\mathbf{F}\mathbf{K} = \begin{pmatrix} W_0^0 & & & \\ & \dots & & \\ & & \dots & \\ W_{(K-1)0} & \dots & W_{(K-1)(K-1)} & \\ & & & \dots \end{pmatrix} \quad 1.7$$

$$M = [I_{L \times L} \quad 0_{(K-L) \times L}]^T \quad 1.8$$

where  $I_{L \times L}$  is the identity matrix with the size  $L$ . The channel estimation discussed in this thesis will be based on Eqs. (3.5) and (3.14). For channel estimation, the symbols sending from the transmitter side are agreed by the both sides in advance. These symbols are called pilot symbols.

**DIFFRENT CHANNEL ESTIMATION METHOD**

**LS Channel Estimation**

The least-square (LS) channel estimation method finds the channel estimate  $\hat{H}$  in such a way that the following cost function is minimized

$$J(H) = \|Y - XH\|^2 = (Y - XH)^H(Y - XH) \quad 1.9 = Y^H Y - Y^H XH - H^H X^H Y + H^H H^H XH$$

Derivative faction with respect t

$$\frac{\partial J(H)}{\partial(H)} = -2 (X^H Y)^* + 2 (X^H XH)^* = 0 \quad 2.0$$

We have  $X^H XH = X^H Y$  which gives the solution to the LS channel estimation as

$$\hat{H}_{LS} = (X^H X)^{-1} X^H Y = X^{-1} Y \quad 2.1$$

$$H_{LS}[K] = \frac{Y[K]}{X[K]}, K = 0, 1, 2, \dots, N - 1 \quad 2.2$$

The mean-square error (MSE) of this LS channel estimate is given as

$$\begin{aligned} \text{MSE}_{LS} &= E\{(H - \hat{H}_{LS})^H (H - \hat{H}_{LS})\} \\ &= E\{(H - X^{-1} Y)^H (H - X^{-1} Y)\} \\ &= E\{(X^{-1} Z)^H (X^{-1} Z)\} \quad 2.3 \\ &= E\{Z^H (X X^H)^{-1} Z\} \\ &= \sigma_z^2 / \sigma_x^2 \end{aligned}$$

Note that the MSE in Equation (3.19) is inversely proportional to the SNR  $\sigma_z^2 / \sigma_x^2$ , which implies that it may be subject to noise enhancement, especially when the channel is in a deep null. Due to its simplicity, however, the LS method has been widely used for channel estimation.

**MMSE Channel Estimation**

Consider the LS solution in Equation (2.3)  $\hat{H}_{LS} = X^{-1} Y$ . Using the weight matrix  $W$ , define  $\hat{H} = W \tilde{H}$  which corresponds to the MMSE estimate. Referring to Figure 1.2, MMSE of the channel estimate  $\hat{H}$  is given a

$$J(\hat{H}) = E\{\|e\|^2\} = E\{\|H - \hat{H}\|^2\} \quad 2.4$$

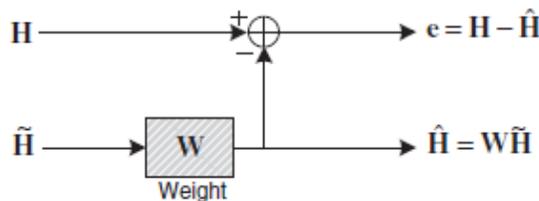


Figure 2 MMSE channel estimation

Then, the MMSE channel estimation method finds a better (linear) estimate in terms of  $W$  in such a way that the MSE in Equation (3.20) is minimized. The orthogonality principle states that the estimation error vector  $e = H - \hat{H}$  is orthogonal to  $\tilde{H}$  such that

$$\begin{aligned}
 E\{e\hat{H}^H\} &= E\{(H - \hat{H})H^H\} \\
 &= E\{(H - W\hat{H})H^H\} & 2.6 & = E\{(H\hat{H}^H - W\hat{H}\hat{H}^H)\} \\
 &= R_{H\hat{H}} - WR_{\hat{H}\hat{H}} = 0
 \end{aligned}$$

where  $R_{AB}$  is the cross-correlation matrix of  $N \times N$  matrices  $A$  and  $B$  and  $\hat{H}$  is the LS channel estimate given a  $= X^{-1}Y = H + X^{-1}Z = R_{H\hat{H}} \cdot R_{H\hat{H}}^{-1}$

$$\begin{aligned}
 R_{\hat{H}\hat{H}} &= E\{H\hat{H}^H\} \\
 &= E\{X^{-1}Y(X^{-1}Y)^H\} \\
 &= E\{(H + X^{-1}Z)(H + X^{-1}Z)^H\} \\
 &= E\{(H\hat{H}^H + X^{-1}ZH^H + HZ^H(X^{-1})^H + X^{-1}ZZ^H(X^{-1})^H)\} \\
 &= E\{(H\hat{H}^H + E\{X^{-1}ZZ^H(X^{-1})^H\})\} \\
 &= E\{(H\hat{H}^H) + (\sigma_z^2 / \sigma_x^2)I\} & 2.7 &
 \end{aligned}$$

$R_{H\hat{H}}$  is the cross-correlation matrix between the true channel vector and temporary channel estimate vector in the frequency domain. Using Equation (2.7), the MMSE channel estimate follows as

$$\begin{aligned}
 \hat{H} &= W\hat{H} = R_{H\hat{H}} \cdot R_{H\hat{H}}^{-1} \hat{H} \\
 &R_{H\hat{H}}(R_{H\hat{H}} + (\sigma_z^2 / \sigma_x^2)I)^{-1} \hat{H} & 2.8 &
 \end{aligned}$$

The elements of  $R_{H\hat{H}}$  and  $R_{H\hat{H}}$  in Equation are

$$E\{h_{kj} \rightarrow h_{kf}\} = \{h_{kj} \rightarrow h_{kf}^*\} = r_f[k-k'] r_f[I-I']$$

Where  $k$  and  $l$  denote the subcarrier (frequency) index and OFDM symbol (time) index, respectively.

**LS Estimator with 1D Interpolation**

1D interpolation is used to estimate the channel at data subcarriers, where the vector  $\hat{H}_{LS}^p$  with length  $Np$  is interpolated to the vector with length  $N$ , without using additional knowledge of the channel statistics. The 1D interpolation methods are summarized in the remainder of this section

**Linear Interpolation (LI)**

The LI method performs better than the piecewise-constant interpolation, where the channel estimation at the data  $\hat{H}_{LS}^p(k)$  and  $\hat{H}_{LS}^p(k + 1)$  given by

$$(sk + 1) = \hat{H}_{LS}^p(k) + \hat{H}_{LS}^p(k + 1) - \hat{H}_{LS}^p(k) \left(\frac{t}{s}\right) \quad 0 \leq t \leq S$$

**Second-Order Interpolation (SOI)**

The SOI method performs better than the LI method, where the channel estimation at the data subcarrier is obtained by weighted linear combination of the three adjacent pilot estimates

**Low-Pass Interpolation (LPI)**

The LPI method is performed by inserting zeros into the original sequence and then applying a low-pass finite-length impulse response (FIR) filter, which allows the original data to pass through unchanged. This method also interpolates such that the mean-square error between the interpolated points and their ideal values is minimized.

**Channel Estimation with SBEM**

Assume the current cell is allocated  $\tau < K$  orthogonal training sequences of length  $L < T$ , where  $T$  is the channel coherence interval, for both uplink and downlink training. Denote the corresponding orthogonal training set in the considered cell as  $S_{cell} = \{S_1, \dots, S_T\}$  with  $S_i^H S_j = L\sigma_p^2 \delta(i - j)$   $\sigma_p^2$  is the pilot signal training power. We propose a new uplink/downlink transmission framework that utilizes the spatial signatures to realize the orthogonal training and data transmission among different users. It is worth mentioning that based on the proposed SBEM, once the spatial signatures of users are obtained in the preamble, the reduced-dimensional channels can be estimated through traditional linear LS method. However, to make the proposed strategy complete and address some ideas in detail, we will generally show the whole procedures of proposed channel estimation

**CONCLUSION**

The aim of this work presented here was to analysis the performance of transmitted signal by combining OFDM with MIMO system. This focuses on to reduce BER via using various parameters. Many algorithm SBEM, Spine Interpolation, Linear Interpolation, LS and MMSE are used here shows graph of OFDM system over known channel. We solve this challenge by designing three different pilot patterns to estimate the wireless channel. A way for enhancing the spectrum utilization efficiency is by improving the bit rate or BER of the wireless system. It has been shown that the MIMO concept is a good means to do that. But then another challenge faces us. The pilot patterns must be redesigned to be suitable for implementation in MIMO OFDM.

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