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THREE-ECHELON SYNCHRONIZED SUPPLY CHAIN DETERMINISTIC INVENTORY MODELS WITH THE INCLUSION OF EMISSION COST

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ABSTRACT

Supply Chain Management plays a major role nowadays in the business world. In today's globalized economy, the supply chain coordination becomes more fruitful in order to minimize the total supply chain cost and thereby increasing the profit. A well integrated inventory management is a key to success of the supply chain system. If there is no proper coordination involved in the system then the overall gain cannot be achieved by the players of supply chain. Environmental problems are an area of steadily increasing concern due to globalization. Carbon emission is one of the notable problems harmful to society. This takes an important place while modelling the supply chain. Carbon emissions are mainly generated in the production process as well as during shipment of products. This paper proposes the study of three echelon closed loop supply chain for two models consisting of four players; a supplier, a manufacturer and a remanufacturer for a single retailer. Emission cost due to transportation is also considered. In the first model, batches of products are received simultaneously from the manufacturer and remanufacturer by the retailer whereas in the second, products are received alternatively. Numerical example is provided to illustrate the proposed two models.

Keywords-*Three echelon supply chain, remanufacturing, emission, environment, shipments, coordination, inventory.*

I. INTRODUCTION

Inventory management has been a focal research area in Operations Research, production and operations management and industrial engineering for many years. The first mathematical treatment of inventory systems was the Economic Order Quantity (EOQ) model developed by Harris in the year 1913. Further major advances in understanding inventory problems took place.

Supply chain coordination decision making can be either centralized or decentralized. Decentralized decisions aim at coordinating various decisions by each participant within the chain leading to conflict of individual objectives. On the other hand, a centralized decision allows a single decision maker, usually consisting of a team, to manage chain as a whole to achieve the overall objective that is collectively optimal for all the entities with the chain. With the growing focus on sustainable supply chain management, firms realize that inventories across the entire supply chain can be more efficiently managed through greater cooperation and better coordination. The collaborative paradigm in supply chain management is a crucial source of competitive advantage, as it can increase the impact and efficiency of the supply chain. The goal of supply chain is to minimize the costs thereby increasing the efficiency of logistics process of the product itself.

For the past few years, remanufacturing has gained considerable attention of researchers and industry practitioners to investigate the most sustainable practices such as reducing manufacturing and energy cost. Remanufacturing is rapidly emerging as an important form of waste prevention and environmentally conscious manufacturing. It is a foundation of recoverable manufacturing system. Material recovery rate is defined as how often a part is in a suitable condition to be remanufactured; any parts which are not recoverable must be replaced by new parts.

Studies that deal with environmental issues in inventory systems are progressively increasing in number. During the past few years, the global economy has evolved considerably. Meanwhile, the increasing GHG emissions have caused a disastrous impact on our society and environment. The greater the distance travelled, the higher the cost of transportation and the bigger the impact on the environment. CO₂ emissions are estimated to be responsible for half of the man made greenhouse effect. Typically, the production inventory problem deals with the decision of manufactured quantity, order, size and inventory level. Such decisions may potentially have significant impact on carbon emission.

The rest of the paper is structured as follows: Section 2 presents the literature review. Section 3 provides fundamental assumptions and notations. Section 4 describes the formulation of two models. Section 5 illustrates a numerical example. Section 6 concludes the paper. A list of references is also provided.

II. LITERATURE REVIEW

There are vast number of papers that addresses the area of production inventory decision making problems along with the recovery of returned material. Schrady (1967) [25] developed a deterministic model to find EOQ for a repairable inventory system considering one manufacturing batch and atleast one repairable batch. He assumed that manufacturing and recovery rates are instantaneous and there is no shortage and disposal cost. The work of Schrady (1967) [25] was extended by Nahmias and Rivera (1979) [19] assuming finite repair rate and limited storage space. Mabini, Pintelon and Gelders (1992) [15] generalized the work of Schrady (1967) [25] by considering backorders. Teunter (2001) [27] extended the model of Schrady (1967) [25] by considering different holding costs for manufactured and recovered products and allowed some quantity of the returned products to be disposed off and also considered the multiple cycles of manufacturing and repairing which are induced alternatively. Choi, Hwang and Koh (2007) [4] developed the work of Teunter (2001) [27] considering the sequence of purchase orders, manufacturing and remanufacturing setups in a production cycle as decision variables. Koh et al (2002) [12] established the work of Teunter (2001) [27] by assuming the recovery rate to be both less and greater than the demand of the serviceable product and they developed models for the policies; one order for new product and many setups for recovery(1,R) and many orders for new product and one setup for recovery(P,1). For applying these models in a practical environment, Teunter (2004) [28] suggested a simple heuristic method to modify the obtained optimal lot sizes of manufacturing and recovery batches in order to assign the integer value to P and R to provide a near optimal solution.

Dobos and Ritcher (2004) [6] analysed a production recycling model where disposal is allowed and they proved that one of the pure strategies is optimal. Konstantaras and Papachristos (2006) [13] generalized both the models of Teunter (2004) [28] by allowing complete backordering. Konstantaras and Papachristos (2008) [14] further enhanced the models of Teunter(2004) [28] by proposing an exact solution method to find integer values of P and R. In the previous models number of manufacturing and remanufacturing cycles are the decision variables along with the lot size of manufactured and remanufactured products. Ritcher and Dobos (1999) [21]; Dobos and Ritcher (2004) [6] studied the return rate and waste disposal rate of the returned product as a decision variables. Teunter and Van Der Laan (2002) [29]; Corbacioglu and Van Der Laan (2007) [5] considered minimizing the discounted total cost as an objective function. While Nikolaidis (2009) [20]; Jena and Sarmah (2014) [11] addressed the issue of acquisition management and pricing policies of the returned products.

Studies of Jaber and El Saadany (2009) [10]; Hasanov, Jaber and Zolfaghari (2012) [9] treated quality of the recovered products different from the manufactured product. Tsai (2012) [30] examined the learning effects on remanufacturing and manufacturing process. Quantitative models in reverse logistics system design is investigated by Fleischmann et al (1997) [7]. Guide, Jayaraman and Srivastava (1999) [8] studied about the production planning and control in remanufacturing. Akcali and Cetinkaya (2011) [1] addressed the inventory control and production planning models related to closed loop supply chain. Sasikumar and Kannan (2008a [22]; 2008b [23]; 2009 [24]) investigated about the various issues in reverse supply chain. Mitra (2009) [16] proposed a generalized inventory management problem considering one depot and one distributor for a single retailer in the case of two echelon supply chain. Works of (Mitra (2009) [16]; Teng et al (2011) [26]; Mitra (2012) [17]) focussed on two echelon closed loop supply chain considering more than one player.

This paper is an extension of “**Two-echelon closed-loop supply chain deterministic inventory models in a batch production environment**” by Bimal Kumar Mawandiya, J.K.Jha & Jitesh Thakkar. In this paper synchronization of three echelon closed loop supply chain consisting of a supplier, a manufacturing and a remanufacturing player for a single retailer is investigated to determine the optimum lot sizing and shipment policy for the minimum joint total cost of the system. Emission cost due to transportation is also considered along with the joint total cost.

III. NOTATIONS AND ASSUMPTIONS

The following notations and assumptions are used to establish the mathematical models.

Notations

μ	demand per unit time on the retailer	
r	fraction of demand returned per unit time, $0 < r < 1$	
α	conversion factor of the returned product to the remanufactured product, $0 < \alpha \leq 1$	
$r\alpha$	fraction of demand which is remanufactured per unit time	
P	production rate of the manufacturer, $P > \mu(1 - \alpha r)$	
Q	lot size of the product received in one replenishment cycle at the retailer from the manufacturer and the remanufacturer (decision variable)	
A_1	Cost per order of the retailer (\$/order)	
A_2	Cost per production setup of the manufacturer (\$/setup)	
A_3	Cost per production setup of the remanufacturer (\$/setup)	
A_s	Cost per order of the supplier (\$/order)	
h_1	inventory holding cost per unit per unit time of the retailer (\$/unit/time)	
h_2	inventory holding cost per unit per unit time for the finished product of the manufacturer (\$/unit/time)	
h_3	inventory holding cost per unit per unit time for the returned material of the remanufacturer (\$/unit/time)	
h_s	inventory holding cost per unit per unit time of the supplier (\$/unit/time)	
k	interest rate (\$/year)	
β	social cost from vehicle emission (mu/h)	
d	distance travelled (from supplier to buyer, km)	
v	average velocity (km/hr)	
m_j	number of shipments made to the retailer during one production cycle of the manufacturer and the supplier in Model- j ($j = 1, 2$), where m_j is a positive integer (decision variable).	

Assumptions

The assumptions of this model are that

- The retailer uses the remanufactured product which is good as the new product to satisfy the demand of customers. Consumable items required for recovery are not covered in the models.
- The demand and return rates are assumed to be deterministic, stationary and uniform over infinite time horizon.
- Remanufacturing of returned products occur in batches and the time taken is negligible.
- Rejected material is discarded immediately during remanufacturing.
- Shortages are not permitted.

IV. MODEL FORMULATION

A three echelon closed loop supply chain is considered with a supplier, a manufacturer and a remanufacturer for a single retailer. The customer’s demand was satisfied by the retailer through recovered and new products received from the remanufacturer and the manufacturer respectively where the raw materials are supplied by the supplier. The retailer receives a lot of quantity $(1 - \alpha r)Q$ from the manufacturer and a quantity of αrQ from the remanufacturer. The objectives of the models are to determine the optimal production inventory policy of the players for minimizing the joint total cost of the system.

Model-I (case of concurrent replenishment policy at the retailer)

In this model, the lots from the manufacturer and the remanufacturer are received simultaneously at the retailer where the raw materials are supplied by the supplier. The cycle length of the retailer and the remanufacturer is Q/μ . The production cycle length of the supplier and the manufacturer is $m_1 Q/\mu$.

The retailer’s average inventory per unit time is $Q/2$.

The manufacturer’s average inventory per unit time is

$$(1 - \alpha r)[(m_1 - 1) - (m_1 - 2)(1 - \alpha r)\mu/P] Q/2 .$$

The remanufacturer’s average inventory per unit time is $rQ/2$.

The supplier’s average inventory per unit time is $(m_1 - 1)k Q/2$.

Emission cost due to transportation is $2\beta \frac{d}{v}$.

Therefore, the joint total cost per unit time of the closed loop supply chain of Model-I is the sum of the ordering and the inventory holding costs per unit time of the retailer and the supplier, production set up and inventory holding costs per unit time of the manufacturer and the remanufacturer along with the emission cost is given by

$$JTC(Q, m_1) = \left(A_1 + \frac{A_2}{m_1} + A_3 + \frac{A_s}{m_1} + \frac{2\beta d}{vm_1} \right) \frac{\mu}{Q} + [h_1 + (1 - \alpha r) \left\{ (m_1 - 1) - (m_1 - 2)(1 - \alpha r) \frac{\mu}{P} \right\} h_2 + rh_3 + (m_1 - 1)h_s k] \frac{Q}{2} \tag{1}$$

To find the optimal value of Q for fixed value of m_1 , the first partial derivative of $JTC(Q, m_1)$ is set to zero and the optimal value of Q is given by,

$$Q^*(m_1) = \sqrt{\frac{2\mu \left(A_1 + \frac{A_2}{m_1} + A_3 + \frac{A_s}{m_1} + \frac{2\beta d}{vm_1} \right)}{h_1 + (1 - \alpha r) \left\{ (m_1 - 1) - (m_1 - 2)(1 - \alpha r) \frac{\mu}{P} \right\} h_2 + rh_3 + (m_1 - 1)h_s k}} \tag{2}$$

Now after substituting the value of $Q^*(m_1)$ from (2) in (1), the joint total cost is expressed as

$$JTC(m_1) = \sqrt{2\mu \left(A_1 + \frac{A_2}{m_1} + A_3 + \frac{A_s}{m_1} + \frac{2\beta d}{vm_1} \right) \left[h_1 + (1 - \alpha r) \left\{ (m_1 - 1) - (m_1 - 2)(1 - \alpha r) \frac{\mu}{P} \right\} h_2 + rh_3 + (m_1 - 1)h_s k \right]} \tag{3}$$

Let $F(m_1) = [JTC(m_1)]^2$

Therefore,

$$F(m_1) = 2\mu \left(A_1 + \frac{A_2}{m_1} + A_3 + \frac{A_s}{m_1} + \frac{2\beta d}{vm_1} \right) \left[h_1 + (1 - \alpha r) \left\{ (m_1 - 1) - (m_1 - 2)(1 - \alpha r) \frac{\mu}{P} \right\} h_2 + rh_3 + (m_1 - 1)h_s k \right]$$

To obtain the optimal value of m_1 , $F(m_1)$ is differentiated with respect to m_1 and set to zero. The optimal value of m_1 is given by

$$m_1^{**} = \sqrt{\frac{\left(A_2 + A_s + \frac{2\beta d}{v} \right) \left[h_1 - (1 - \alpha r) \left\{ 1 - 2(1 - \alpha r) \frac{\mu}{P} \right\} h_2 + rh_3 - h_s k \right]}{(A_1 + A_3) \left[(1 - \alpha r) \left\{ 1 - (1 - \alpha r) \frac{\mu}{P} \right\} h_2 + h_s k \right]}} \tag{4}$$

Model-II(case of alternate replenishment policy at the retailer)

In this model the lots of products are received from the manufacturer and remanufacturer not at the same time by the retailer in which the raw materials are supplied by the supplier. The cycle length of the retailer and the remanufacturer is Q/μ . The production cycle length of the supplier and the manufacturer is m_2Q/μ .

The retailer’s average inventory per unit time is $[(ar)^2 + (1 - ar)^2] Q/2$.

The manufacturer’s average inventory per unit time is

$$(1 - ar)[(m_2 - 1) - (m_2 - 2)(1 - ar) \mu/P] Q/2.$$

The remanufacturer’s average inventory per unit time is $rQ/2$.

The supplier’s average inventory per unit time is $(m_2 - 1)k Q/2$.

Emission cost due to transportation is $2\beta \frac{d}{v}$.

Therefore, the joint total cost per unit time of the closed loop supply chain of Model-II is the sum of the ordering and the inventory holding costs per unit time of the retailer and the supplier, production set up and inventory holding costs per unit time of the manufacturer and the remanufacturer along with the emission cost is given by

$$\begin{aligned}
 JTC(Q, m_2) = & \left(A_1 + \frac{A_2}{m_2} + A_3 + \frac{A_s}{m_2} + \frac{2\beta d}{vm_2} \right) \frac{\mu}{Q} \\
 & + [\{ (ar)^2 + (1 - ar)^2 \} h_1 + (1 - ar) \{ (m_2 - 1) - (m_2 - 2)(1 - ar) \mu/P \} h_2 + rh_3 \\
 & + (m_2 - 1)h_s k] \frac{Q}{2}
 \end{aligned}
 \tag{5}$$

To find the optimal value of Q for fixed value of m_2 , the first partial derivative of $JTC(Q, m_2)$ is set to zero and the optimal value of Q is given by,

$$Q^*(m_2) = \sqrt{\frac{2\mu \left(A_1 + \frac{A_2}{m_2} + A_3 + \frac{A_s}{m_2} + \frac{2\beta d}{vm_2} \right)}{\{ (ar)^2 + (1 - ar)^2 \} h_1 + (1 - ar) \{ (m_2 - 1) - (m_2 - 2)(1 - ar) \mu/P \} h_2 + rh_3 + (m_2 - 1)h_s k}}
 \tag{6}$$

Now after substituting the value of $Q^*(m_2)$ from (6) in (5), the joint total cost is expressed as

$$JTC(m_2) = \sqrt{2\mu \left(A_1 + \frac{A_2}{m_2} + A_3 + \frac{A_s}{m_2} + \frac{2\beta d}{vm_2} \right) \left[\frac{\{ (ar)^2 + (1 - ar)^2 \} h_1 + (1 - ar) \{ (m_2 - 1) - (m_2 - 2)(1 - ar) \mu/P \} h_2}{+rh_3 + (m_2 - 1)h_s k} \right]}
 \tag{7}$$

Let $F(m_2) = [JTC(m_2)]^2$

Therefore,

$$F(m_2) = 2\mu \left(A_1 + \frac{A_2}{m_2} + A_3 + \frac{A_s}{m_2} + \frac{2\beta d}{vm_2} \right) \left[\{(\alpha r)^2 + (1 - \alpha r)^2\} h_1 + (1 - \alpha r) \left\{ (m_2 - 1) - (m_2 - 2)(1 - \alpha r) \frac{\mu}{P} \right\} h_2 + r h_3 + (m_2 - 1) h_s k \right]$$

To obtain the optimal value of m_2 , $F(m_2)$ is differentiated with respect to m_2 and set to zero. The optimal value of m_2 is given by

$$m_2^* = \sqrt{\frac{(A_2 + A_s + \frac{2\beta d}{v}) \{ (\alpha r)^2 + (1 - \alpha r)^2 \} h_1 - (1 - \alpha r) \left\{ 1 - 2(1 - \alpha r) \frac{\mu}{P} h_2 + r h_3 - h_s k \right\}}{(A_1 + A_3) \left[(1 - \alpha r) \left\{ 1 - (1 - \alpha r) \frac{\mu}{P} \right\} h_2 + h_s k \right]}} \quad (8)$$

V. NUMERICAL EXAMPLE

In this section we provide the numerical example to illustrate the models developed in the previous section. The following parameters are used for finding the result:

$$\mu = 10,000, P = 15,000, A_1 = 100, A_2 = 400, A_3 = 200, A_s = 200, h_1 = 40,$$

$$h_2 = 20, h_3 = 10, h_s = 70, r = 0.25, \alpha = 0.9, \beta = 0.5, d = 250, v = 180, k = 15\%.$$

Hence $m_1^* = 2, Q^* = 419$ and $JTC(Q, m_1) = \$28687$ for Model I and

$$m_2^* = 1, Q^* = 702 \text{ and } JTC(Q, m_2) = \$25672 \text{ for Model II.}$$

VI. CONCLUSION

The integrated three layer supply chain model comprising of a supplier, a manufacturing and a remanufacturing player for a single retailer along with the emission cost due to shipments is presented in this paper. This paves the way to reduce the total cost of the supply chain so that each of the player will be benefitted. Finally, a numerical example is provided to demonstrate its practical usage..

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