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AN OPTIMAL INVENTORY POLICY FOR MONOPOLISTIC MARKET EXPERIENCING HYBRID BACKORDER AND SWITCHING COST

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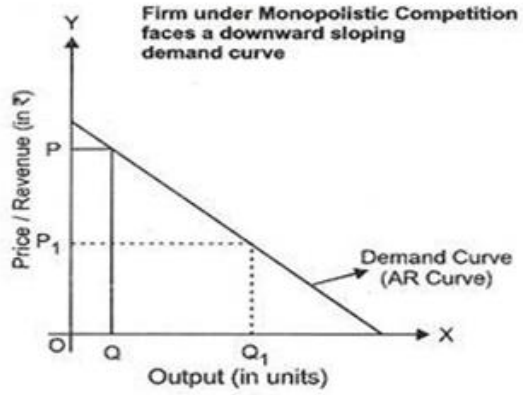
ABSTRACT

In the present day, companies compete with one another to attain the peak of achievement. The products that are sold in monopolistic and captive market need to follow optimal strategies to become successful. Monopolistic competition is that many manufacturers sell goods that are differentiated from one another by branding or quality. This paper introduces inventory into the model of monopolistic competition, which undergoes hybrid backorder. It is fitting to take into consideration of transportation cost in the model since the goods should be delivered at the right time and place. A distinctive aspect of the model is that the cost of advertising and brand recognition is included since monopolistic market needs publicity. Backorder price discount is offered to avoid the lost sales. A numerical example is provided to exemplify the proposed model.

Keywords- *Inventory, Backorder, Switching cost, Transportation, Advertising, Monopolistic market.*

I. INTRODUCTION

Advances in information technology have made it conceivable to track sales and inventory, as well as adjust production and pricing levels, more quickly than ever. Many industries, such as airlines, hotels, and different retailers, use dynamic pricing to match demand with capacity or inventory, maximize revenue or accomplish other key objectives. The theory of monopolistic rivalry was progressed autonomously seven decades ago on different sides of the Atlantic by Edward Chamberlin [3] and Joan Robinson [19]. Their key contribution was to recognize that even when there are a large number of firms in an industry, the demand curve facing each firm will be downward sloping because of product differentiation, just like monopoly.



But at the same time, the theory encompasses the case in which there is free entry and exit into the industry, which means that economic profit will be driven in the limit to zero, just like pure competition. Dhingra and Morrow [6] fully characterized the optimality properties of a general demand system derived from separable group utility. The validity of these insights on international trade issues is discussed by Zhelobodko, Kokovin, Parenti, and Thisse[26]. Advertising is vital for economic growth of a firm. It is generally condemned that the cost of advertisement is higher than the selling price of the advertising goods. They feel that it is an additional expense, but in this modern era of high competition advertising informs the buyers about the capacity of a certain product or service in the market and promotes them to buy it .Some vibe that advertisement influences the mind of the consumers .The proprietors of

small firms are disappointed as they cannot bear equivalent amount of money as the larger firms do. Advertising results in monopoly that makes the consumer become attracted to a particular brand. Advertising is a powerful educational tool able of reaching and motivating the customers. Huang, Li [10], Szmerekovsky and Zhang [22] extended the work of Huang and Li [10] in developing a price discount model to coordinate the advertising expenses of the two parties.

Paying notice to delivery lot size would be better to consider transportation as well. It is due to more than 50% of total annual logistic costs can be pointed towards transportation that is also known as freight cost [16]. In this paper freight costs is the function of shipping weight and distance. Economic lot size models have been studied extensively since Harris [9] presented the famous EOQ formula in 1913. Five years later, the economic production quantity (EPQ) inventory model was proposed by Taft (1918) [7]. Both models have been extensively used by industries and enterprises in the management of their inventories. Roach [2] presents an interesting historical discussion on the likely derivation of Harris's EOQ model. Baumol and Vinod [24] research work was the first introduced inventory theoretic models as the integration of transportation and inventory costs. Swenseth and Godfrey [16] hence studied the effect of joining freight function into an EOQ model for inventory replenishment decision. Chen and Sarker [25] revised that freight cost becomes the function of shipping weights and distances. It is due to the prior freight rates proposed by Swenseth and Godfrey [16] only focused upon the shipping weight, whereas distance assumed to be fixed. Srivastav and Agrawal [20] used weighted MOCS algorithm to optimize a bi-objective backorder inventory model. Mousavi, Niaki, Bahreininejad, andMusa [13] developed a bi-objective multi-item inventory model considering shortages as time dependent backorders with lost sales. Backordering policy provides a better cost control of the inventory system. Various research papers have been published regarding backorder. Hayek and Salameh [15] derived an optimal EPQ policy with rework and imperfect quality items. They assumed that shortage backorders were permitted. Wang [17] developed an inventory problem for deteriorating items with time-varying demands and shortages over a finite planning horizon. He assumed backlogging rate to be time-dependent. Wee et al.(2008) [8] and Pentico et al (2009) [4] discussed about inventory model for deteriorating items with partial backordering under inflation. In 2011 Carrdenas-barron [11] developed an EOQ/EPQ model with backorder costs using analytical geometry and algebra. Srivastav, Sunil Agrawal [21] developed a multi-objective optimization of hybrid backorder inventory model.

Brand awareness is essential for any company especially that is building an online reputation. Generally, companies keep marketing and branding budget between 2 to10 percent of their sales. Marketing professionals will be able to explain the world of branding. Creating an effective brand awareness strategy without spending a lot will lead the firm to be successful. An influential source for marketers is word of mouth communication, also known as buzz marketing. This is customer generated and viral marketing for businesses and this does not cost a thing. In some cases of buzz marketing, business is known to hand out product samples and offer their goods to famous, influential people, so that they can suggest and promote others to buy their brand. For example, pharmaceutical companies will get physicians to talk up their products with a purpose of persuading their peers to buy their products[1].Viral buzz – In this present world media and networks are becoming a real phenomenon, so marketers are trying to concentrate their advertising here and publicize their brand using the likes of Facebook, Snapchat, YouTube, etc. [1].

Brand switching is a situation in which somebody changes from buying one brand of a product to buying a different brand. Companies often use price promotions to promote brand switching. Brand switching can be activated by price promotions, in-store displays, superior availability, perceived improvements or modernization in competitive brands, desire for novelty, number of available brands, perceived risk, frequency of purchase, changes in quality, or level of satisfaction with the most recent purchase. Brand switching is generally usual with products that have no great perceived variant in quality across brands such as bottled water, dairy products, or paper towels. But unexpected brand switching can cost the firm a lot. This study constitutes an effort to develop the model of monopolistic inventory by incorporating transportation cost, advertising cost, brand switching cost, brand recognition cost and finally in order to avoid lost sales, discount on backorder is offered.

The rest of the paper is organized as follows. The upcoming Section 2 shows the notations and assumptions of the model considered in the paper. In Section 3 mathematical formulation of the proposed inventory model is discussed. Section 4 presents a numerical example. Lastly, Section 5 concludes the proposed work.

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II. LITREATRE REVIEW

Notations

A	-	Fixed set up cost per order
D	-	Annual demand
h	-	Holding cost per unit per unit time
π	-	Backorder cost per unit short
π_b	-	Backorder cost per unit backorder per unit time
Q	-	Order quantity
μ	-	Lead time demand
σ	-	Standard deviation of lead time demand safety
K	-	Safety stock factor
$G_u(K)$	-	A special function of unit Normal (mean 0, standard deviation 1) and is used in finding in expected shortage per replenishment cycle
P(u>K)	-	Stock out risk (tail probability)
S	-	Reorder point
B(S, μ , σ)	-	$\int_S^\infty (x-S)\left[1-F(x)\right]dx$
C	-	Unit cost of item
I	-	Carrying charge
α	-	Discount factor for LTL shipments
F_x	-	The freight rate for full truckload product.
W_x	-	Full truckload shipping weight
d	-	Transportation distance
w	-	Weight of a unit part
I_C	-	Initialisation cost
n	-	Number of times the advertisement is telecasted
t_c	-	Cost of telecasting the advertisement once
B_s	-	Brand switching cost
P_r	-	Marketing professional cost
V_r	-	Viral buzz
B_Z	-	Buzz marketing
β_o	-	Upper bound of the backorder ratio
π_x	-	Back order price discount offered by the vendor per unit
π_o	-	Cost of lost demand per unit
TC	-	Total annual inventory cost
*	-	Corresponding optimal values

Assumptions

1. The lead time is known and constant.
2. Unfilled demand is completely backlogged.
3. Stock out cost is considered as combination of fixed backorder cost (per unit short) and time-weighted backorder cost (per unit backorder per unit time) [21].
4. The reorder point is non-negative.
5. The demand during the lead time follows the Normal distribution.
6. The initialisation cost of casting an advertisement is the amount of money spent by the firm to get in connection with the advertising agency.
7. The shortages are combination of fixed and time proportional backorders

III. MATHEMATICAL FORMULATION

The total inventory cost is the sum of ordering cost, holding cost, backorder cost for both fixed and time proportional, transportation cost, advertising cost, brand switching cost, brand recognition cost and with a discount offered on backorder.

$$Minimize\ TC\ (Q,\ K) = \frac{AD}{Q} + h\left(\frac{Q}{2} + K\sigma\right) + \frac{\pi D}{Q}\sigma G_u(K) + C\left(1 - \frac{\beta_O \pi_x}{\pi_O}\right) + \frac{(h+\pi_b)B(S,\mu,\sigma)}{Q} + \frac{D}{Q}\alpha F_x W_x d + Ddw(1-\alpha)F_x + \frac{(I_c + nt_c)D}{Q} + \frac{B_s D}{Q} + (P_r + V_r + B_z)\frac{D}{Q}$$

---- (1)

The time proportional backorder component can be written from [5]

$$B(s,\mu,\sigma) = \sigma^2[a_2 G^2{}_u(k) + b_2 G_u(k) + c_2]$$ ----- (2)

where, a_2= 0.479706, b_2= 0.443603 and c_2= −0.001984 (can be removed)

Substituting (2) in (1) we get,

$$Minimize\ TC\ (Q,\ K) = \frac{AD}{Q} + h\left(\frac{Q}{2} + K\sigma\right) + \frac{\pi D}{Q}\sigma G_u(K) + C\left(1 - \frac{\beta_O \pi_x}{\pi_O}\right) + (P_r + V_r + B_z)\frac{D}{Q} + \frac{(h+\pi_b)\sigma^2[a_2 G^2{}_u(k) + b_2 G_u(k)]}{Q} + \frac{D}{Q}\alpha F_x W_x d + Ddw(1-\alpha)F_x + \frac{(I_c + nt_c)D}{Q} + \frac{B_s D}{Q}$$

---- (3)

The above expression is convex and it is given by

Let TC (Q, K) $_1 = \frac{AD}{Q} + h\left(\frac{Q}{2} + K\sigma\right)$

$$\frac{\partial TC\ (Q,\ K)_1}{\partial Q} = \frac{-AD}{Q^2} + \frac{h}{2},\ \frac{\partial^2 TC\ (Q,\ K)_1}{\partial Q \partial K} = 0$$

$$\frac{\partial^2 TC\ (Q,\ K)_1}{\partial Q^2} = \frac{2AD}{Q^3},\ \frac{\partial TC\ (Q,\ K)_1}{\partial K} = h\sigma,\ \frac{\partial^2 TC\ (Q,\ K)_1}{\partial K^2} = 0$$

The Hessian matrix is positive semi definite and this shows that it is strictly convex. Similarly procedure for the other expressions shows that $TC\ (Q,\ K)$ is convex.

Now differentiating the equation (3) with respect to Q and K we get the corresponding optimal order quantity as follows

$$Q^* = EOQ\sqrt{1 + \frac{H}{A} + \frac{\sigma^2(h+\pi_b)\theta}{AD}}$$ ----- (4)

Where

$$H = \pi\sigma G_u(K) + \alpha F_x W_x d + I_c + nt_c + B_s + P_r + V_r + B_z$$

$$\theta = a_2 G^2{}_u(k) + b_2 G_u(k)$$

Using the tail probability result we get $P(u > K)^* = \frac{hQ}{\pi D + \sigma(h+\pi_b)(2a_2 G_u(k) + b_2)}$ ---(5)

The approximation for estimation of $G_u(k)$for Normal lead time demand in time-weighted backorder model is given by [18] as

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$$G_u(k) = 0.4115\left[1 + Ln\left\{\frac{P(u > K)}{1 - P(u > K)}\right\}\right]$$ where Ln(x) is the natural logarithm of x

IV. NUMERICAL EXAMPLE

Following numerical example is considered to illustrate the proposed inventory model

A	-	15	C	-	50
D	-	700	I	-	0.2
h	-	10	α	-	0.11246
π	-	100	F_x	-	0.0000402174
π_b	-	1500	W_x	-	46000
π_x	-	5	d	-	600
μ	-	300	w	-	22
σ	-	50	I_C	-	100
π_o	-	150	n	-	6
$G_u(K)$	-	-1.6011	t_c	-	1340
P(u>K)	-	0.9914	B_s	-	45
B_Z	-	1500	P_r	-	500
β_o	-	0.5	V_r	-	400

On substituting the above values in the corresponding equations we get,
H= 18715.33, θ = 0.51945

Optimal order quantity **Q* = 84 units**
Safety stock factor **K*= 2.98**
Total inventory cost = **Rs.25, 035.397**

V. CONCLUSION

This paper studies the monopolistic and captive market based inventory model emphasizing hybrid backorder with transportation cost, advertising cost, brand switching cost and brand recognition cost. In order to avoid lost sales discount is offered on backordered items which matches the real market behaviour. Advertisement plays a significant role in today's world. Without the help of advertisement a company cannot earn brand recognition. Since the firm is under monopolistic competition, customer loyalty is very important. Buzz marketing helps the firm to sustain in the monopolistic competition. The advantage of the proposed model is that it helps the firm to estimate the optimal values and maximize the profit.

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