ISSN 2277 – 5528
Impact Factor- 4.015

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES &MANAGEMENT

SALVAGING OF DEFECTIVE ITEMS WITH EPQ MODEL

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ABSTRACT

In a production/inventory situation items which arereceived or produced are not of perfectquality. Thus the presence of defects which is inevitable in a produced/ordered lot is sorted out by the process of screening. Handling of the defective items varies by industry and product types. For example, defective items may be dealt by the way of rework process or by sold at discount prices.We consider an EPQ model with salvaging process of defective items for which the items of poor quality detected in the screening process of a lot are sold at a discounted price and we considered handbill adv. to get the customer's attention about the product and brand name with the inclusion of some environmental issues too. The expected profit functions are developed using renewal reward theory, and closed form of expressions for the optimal production lot size are derived with numerical example

Keywords-Economic Production Quantity, Random yield, defective items, Discount sale

I. INTRODUCTION

We consider a machine producing a single item, with the possibility of producing a random proportion of defective items. To identify the defective items, screening is conducted at the end of the production period. Once identified, the defective items are reworked at a constant rate before they are returned to the inventory. A common assumption in the inventory literature with defectives is that the rework of a defective item is followed immediately after it is identified.Handling of defective items varies by industry and product types. For example, defective items may be sold at discount in the apparel industry, or reworked in the automobile industry where the final product is very expensive. To simplify the analysis, it was assumed into two groups, good products and to be reworked products. Our paper, proceeded the lot sizing model with the salvaging process of defective items at discount sale when the production system is very expensive.In this case, it is better to choose discount sales for the defective products than the rework process.

Consumers are always looking for ways to save money, and they prefer to buy from stores that offer the best deals. For example,when we go to shopping and when we see the word discount we get excited because discount means shopper would be saving money on items he or she intends to buy.Offering discounts and special deals is an effective way to gain consumer attention. Depending on how they advertise their discounts.Advertising is the best way to communicate to the customers and it helps to inform the customer about the brand available in the market and the variety of products useful to them. Yet there are lots of ways to advertise, but we considered the advertisement via handbills. Distribution of handbills is also one of the mediums of advertising. The bills are issued to the people for them to get to know the motive ofadvertisement.

This paper elucidates about salvage process; comprises the formulation of an EPQ model with the inclusion of transportation cost, cost of emission from transportation, cost of energy usage, emission from energy usage and cost of advertisement via handbills., presents a numerical example concludes the proposed work.The remainder of the paper is organized as follows, Section 2 presents the literature review of the previous works cited. Section 3 presents model formulations with notations and assumptions which are used in this model, Section 4 gives numerical example and in section 5 concludes the proposed work.

II. LITERATURE REVIEW

Inventory models with imperfect quality items have received significant attention in the literature. For a survey on these models, we refer the following, [17] Considers a production/inventory situation where items, received or produced, are not of perfect quality. In [13] the effect of imperfect quality items on the finite production model, when production stops, the defect items are assumed to be reworked at a constant rate were discussed. An economic production quantity model with random defective rate, rework process and backorders for a single stage production system was derived [2].In [20] examines the finite production model with a random defective rate and the imperfect

ISSN 2277 – 5528
Impact Factor- 4.015

rework process.[5] Investigates an EPQ model for imperfect quality items with non-synchronized screening and rework. Our work is closest to Lama Moussawi-Haidar[8] who analyze two models assumes that defective items are sold at a discount at the end of the production cycle and assumes that defective items are reworked at a constant rate. In [28] EOQ models with a discounted price are developed to obtain the optimal ordering policy during the sale period forfive different cases. [27] Optimizes a Vendor Managed Inventory (VMI) Supply Chain for Perishable Products by Considering Discount by using two Calibrated Metaheuristic Algorithms.A general model is proposed for a single-item lot-sizing problem with multiple suppliers, quantity discounts, and backordering of shortages [29]. [30] Presents to investigate an inventory model for imperfective items under a one-time-only discount, where the defectives can be screened out by a 100% screening process and then can be sold in a single batch by the end of the 100% screening process.

In general, the need for transportation begins, when the retailers and the buyers are far apart from each other. Langley [9] discussed the need for the inclusion of transportation cost to the inventory model, Russell [16] derived the EOQ model with the inclusion of transportation cost of a single item. Carter [3] substantiated the importance of transportation cost in his work. Jaber[11] explicated the requirement of transportation along with the social cost that results from the emission of pollutants. Energy consumption is assumed to occur both during production and during the idle state of the machines [19] and Energy and resource efficiency is becoming an important strategy in manufacturing and it describes clearly in [22]. Also Bazan[1] considers energy used for manufacturing and remanufacturing. [25] Evaluated the efficiency of Internet banner advertisements and in [23] expanded thedemand through price advertisement.

III. MODEL FORMULATION

As in Salameh [8],we consider the production occurs at a rate α, and demand occurs at a rate β units per unit time, $\alpha > \beta$. During production, a random proportion P of defective items is produced, with a known probability function f (P). The number of defective items accumulated at the end of the screening (during & after production) period are sold at a discount price v. To avoid shortages, the number non-defective items produced should be greater or equal to the demand during production. i.e.$N(y,t) \geq \beta t_1$, which implies $P \leq 1 - \frac{\beta}{\alpha}$.
The following notations and assumptions are used to develop the model.

Notations

α	production rate per unit time
β	demand rate of the final product per unit time
c_p	unit production cost
d	production rate of defective items per unit per unit time ,$d = \alpha P$
d_1	screening cost per item during production
d_2	screening cost per item after production stops
h	holding cost per unit per unit time
K	fixed production setup cost
P	random proportion of defective items, with probability density functionf (P)
f (P)	probability density function of P
s	unit selling price of good quality items, $s > v$
t_1	production time ,$t_1 = y/\alpha$
t_2	screening time
T	production cycle
x	screening rate per unit per unit time
y	total number of items produced during a production cycle
c	cost of energy per unit
d_t	distance travelled(from supplier to buyer, km)
a	fixed cost per trip (mu)
b	variable cost per unit transported per distance travelled (mu /unit/km)

ISSN 2277 – 5528
Impact Factor- 4.015

t_c capacity of the vehicle
β_1 social cost from vehicle emission (mu / h)
v_1 average velocity (Km / h)
β_2 social cost of emission from energy usage
A capital cost of adv. through handbills
n number of employees involved in distribution of handbills
z cost per employee
TC(y) total cost per cycle
TP(y) total profit per cycle

Assumptions

- No shortages are allowed.
- Demand during production is met from non-defective items only.
- Defective items are accumulated and sold at a discount price.
- Emissions are considered from transport and energy usage.
- Handbill advertising is considered to develop and to get attention of the customers for a discount sale.

Let TC(y) be the total cost per cycle. TC(y) is the summation of the production setup cost, unit production cost, screening cost during and after production, inventory holding cost, transportation cost, emission cost from transportation, cost of energy usage, emission from energy usage and cost of advertisement via handbills.
As a result the total cost per cycle is written as,

$$TC(y) = K + c_p y + d_1 \frac{\beta}{(1-P)}\frac{y}{\alpha} + d_2 y\left[1 - \frac{\beta}{\alpha} - \frac{P\beta}{\alpha(1-P)}\right] + \left(\frac{2a}{t_c} + bd_t y\right) + 2\beta_1 \frac{d_t}{v_1} + \frac{Cy}{\alpha} + \alpha\beta_2 + (A + nz) + h\left[\frac{y^2\left(1-\frac{\beta}{\alpha}-P\right)^2}{2\beta} + \frac{y^2\left(1-\frac{\beta}{\alpha}\right)}{2\alpha} + \frac{y^2 P\left(1-\frac{\beta}{\alpha}-\frac{P\beta}{\alpha(1-P)}\right)}{x}\right] \quad \text{--- (1)}$$

Let TR(y) be the total revenue per cycle. TR(y) is the summation of the selling price of good quality items and the discounted selling price of defective items. Thus it is written as,

$$TR(y) = sy(1-P) + vyP \quad \text{--- (2)}$$

The total profit per cycle is the total revenue less the total cost, and is given as

$$TP(y) = sy(1-P) + vyP - \left\{\begin{array}{l} K + c_p y + d_1 \frac{\beta}{(1-P)}\frac{y}{\alpha} + d_2 y\left[1 - \frac{\beta}{\alpha} - \frac{P\beta}{\alpha(1-P)}\right] \\ + \left(\frac{2a}{t_c} + bd_t y\right) + 2\beta_1 \frac{d_t}{v_1} + \frac{Cy}{\alpha} + \alpha\beta_2 + A + nz \\ + h\left[\frac{y^2\left(1-\frac{\beta}{\alpha}-P\right)^2}{2\beta} + \frac{y^2\left(1-\frac{\beta}{\alpha}\right)}{2\alpha} + \frac{y^2 P\left(1-\frac{\beta}{\alpha}-\frac{P\beta}{\alpha(1-P)}\right)}{x}\right] \end{array}\right\} \quad \text{--- (3)}$$

Using the renewal – reward theorem [15], the expected profit per unit time is the following

$$ETPU(y) = \frac{ETP\,(y)}{E(T)},$$

Where the expected duration of the production cycle is $E(T) = \frac{y(1-E[P])}{\beta}$. This gives the following expression for the expected profit per unit time,

ISSN 2277 – 5528
Impact Factor- 4.015

$$\mathrm{ETPU}(y) = s\beta + v\beta\frac{E[P]}{(1-E[P])} - \frac{\beta}{y(1-E[P])}\left(K + \frac{2a}{t_c} + 2\beta_1\frac{d_t}{v_1} + \alpha\beta_2 + A + nz\right) - c_p\frac{\beta}{(1-E[P])} - \frac{\beta}{(1-E[P])}\left\{d_1\frac{\beta}{\alpha}E\left[\frac{1}{1-P}\right] + d_2\left(1 - \frac{\beta}{\alpha} - \frac{\beta}{\alpha}E\left[\frac{P}{1-P}\right]\right) + bd_t + \frac{c}{\alpha}\right\} - h\frac{y}{(1-E[P])}\left\{\frac{E\left[\left(1-\frac{\beta}{\alpha}-P\right)^2\right]}{2} + \frac{\beta\left(1-\frac{\beta}{\alpha}\right)}{2\alpha} + \frac{\beta E[P]}{x}\left(1 - \frac{\beta}{\alpha} - \frac{\beta}{\alpha}E\left[\frac{P}{1-P}\right]\right)\right\} \quad \text{--- (4)}$$

The optimal production quantity y^* is obtained by taking the derivative of the expected profit per unit time in (4) and by using the necessary condition $\frac{\partial(\mathrm{ETPU}(y))}{\partial y} = 0$, we get,

$$\frac{\beta}{y^2}\left(K + \frac{2a}{t_c} + 2\beta_1\frac{d_t}{v_1} + \alpha\beta_2 + A + nz\right) = h\left\{\frac{E\left[\left(1-\frac{\beta}{\alpha}-P\right)^2\right]}{2} + \frac{\beta\left(1-\frac{\beta}{\alpha}\right)}{2\alpha} + \frac{\beta E[P]}{x}\left(1 - \frac{\beta}{\alpha} - \frac{\beta}{\alpha}E\left[\frac{P}{1-P}\right]\right)\right\}$$

Hence,the optimal production quantity $\mathbf{y}^*$ has the following expression,

$$y^* = \sqrt{\frac{2\beta\left(K + \frac{2a}{t_c} + 2\beta_1\frac{d_t}{v_1} + \alpha\beta_2 + A + nz\right)}{h\left[E\left[\left(1-\frac{\beta}{\alpha}-P\right)^2\right] + \frac{\beta}{\alpha}\left(1-\frac{\beta}{\alpha}\right) + \frac{2\beta E[P]}{x}\left(1-\frac{\beta}{\alpha}-\frac{\beta}{\alpha}E\left[\frac{P}{1-P}\right]\right)\right]}} \quad \text{--- (5)}$$

IV. NUMERICAL EXAMPLE

In this paper, we consider an example with the following parameters,

β = 1200, α = 1600, C_p = 104, s = 200, x = 1unit/min, d_1= 0.5,d_2 = 0.6, t_c = 50, n = 2,
K = 1500, h = 20,v_1 =20, a = 3, b = 0.5, v = 80, c = 0.2, A = 50, z = 20,β_1= 0.2,
β_2= 0.1, d_t = 25.

Assume that the inventory operation operates on an 8 hours/day, for 365 days a year, then the annual screening rate,x= 1*60*8*365=175200 units/year.Also the proportion of defective items is uniformly distributed over the range [0, 0.1], with the probability distribution function
f (P) as follows,

$$f(P) = \begin{cases} 10\,, \text{for } 0 \le P \le 0.1 \\ 0\,, \quad \text{otherwise.} \end{cases} \quad \text{--- (6)}$$

Using(6),we get the expected value expressions:E [P] = 0.05,$E\left[\frac{1}{1-P}\right] = 1.0536$,

$E\left[\frac{P}{1-P}\right] = 0.0536$, $E\left[\left(1-\frac{\beta}{\alpha}-P\right)^2\right] = 0.0410$.

Then the optimal production quantity of y is derived from Eq. (5), we get$\mathbf{y^* = 958.5}$**units**.Substituting$y^* = 958.5$units in Eq.(4) the expected profit is $\mathbf{ETPU(y^*) = \$\,92{,}622.59/yr}$.

V. CONCLUSION

This paper presents an Economic production quantity model with salvagingof defective items produced by the way of discount sale including the costs of transportation, emission from transportation, energy usage, emission from energy usage and advertisement via handbills. The equations to calculate the optimal total profit per unit time and order quantities are presented. A numerical example is provided to demonstrate its practical usage.

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